An Interview with Alissa S. Crans

Don't be shy about asking lots of questions!

Dr. Alissa S. Crans is an Associate Professor of Mathematics at Loyola Marymount University. She is also the Associate Director of Diversity and Education at the Mathematical Sciences Research Institute.

Ken: How did you become interested in mathematics?

Alissa: I always liked math, so I took as much of it as I could and learned as much as I could. I was very lucky to have teachers that inspired and challenged me. In the 5th grade, Mr. Boots took the time to teach me how to cross multiply (or cross cancel) when multiplying fractions before he taught the rest of the class. In the 6th grade, Mr. Peschman would put problems on our homework and tests specifically to challenge me. Also, my dad was a math major in college, so all throughout elementary and middle school, I would sit down with him after dinner and explain my math homework to him. If I had made a mistake on any problem, he'd ask me to show him how I did it, and then I would see my error and correct it. So for me, mathematics was a very social, fun activity.

In college I starting learning more abstract math and it was even more exciting and fun than the mathematics I had learned previously! I remember when I started learning about infinity and just feeling like my mind was being blown. For example, we were asked whether we thought there are more counting numbers {1, 2, 3, ...} or even counting numbers {2, 4, 6, ...}. Of course we all said counting numbers because there are clearly twice as many of those than the even ones. But we were wrong! These collections of numbers are the same size! You can see this by matching them up with each other: We match 1 with 2, 2 with 4, 3 with 6, 4 with 8, and so on. It was just incredible and, at the time, shocking!

Ken: On your website, you quote the mathematician Sonia Kovalevskaya where she says that a mathematician must have a spirit like that of a poet. Can you please give an example of something in math that illustrates this claim? Where is the poetry in mathematics?

Alissa: I don't think of math as being exactly like poetry, but I like this quote because it likens mathematics to an art. Mathematicians find beauty in their work and are extremely creative when posing and answering questions. We all love what we do and often appreciate mathematics as it is, regardless of whether it has a practical or real-life application. I often ask my students what the practical application is in a painting or sculpture or novel or piece of music. Mathematics is no different – to many people it is beautiful, interesting, and fun on its own and can be enjoyed and appreciated much in the same way that we appreciate and enjoy visiting museums, reading poetry, or attending concerts.

To many people, thinking of math as an art may seem strange because I think that when many people talk about mathematics, they are thinking about arithmetic computation and various formulas and rules they were told to memorize such as the quadratic formula, SOH-CAH-TOA, the Pythagorean theorem and the like. But when I talk about mathematics, I'm thinking about patterns and symmetries – like understanding not just how to use the Pythagorean theorem to compute something, but why it is so and what its implications are. For instance, in my own mathematical research, I rarely perform numerical computations. In fact, I can't remember the last time I wrote down a number while working on my own research!

This brings up another aspect of mathematics. Math is a huge subject! So if you don't particularly like the math you're doing in school, please don't conclude that math just isn't your thing. For example, while many mathematicians love studying the patterns in numbers, there are also lots of mathematical fields that have nothing to do with numbers, like topology, and in particular knot theory, which is one of my main research interests.

Ken: In your research statement, you describe your desire to find definitions for certain concepts, such as "higher-dimensional cocycles." I think the idea of finding the definition of something is new to many K12 students because in K12 math, definitions are generally supplied by a textbook or teacher. Could you please explain this process of defining something? How do you know if you have succeeded?

Alissa: This is an excellent question! Mathematicians are interested in patterns. Thus, if we notice that a particular pattern or concept is popping up in lots of different areas, we decide to give it a name and formal definition. For example, the notion of "rate of change" shows up in many situations such as when looking at population growth, the increase in the world's temperature, the velocity of your car, and the slope of a line. Thus, mathematicians made this observation formal by defining the concept of the "derivative" and this is the fundamental object of study in differential calculus.

How do we know that we have the right definition? Here's one way: Often times we have an instinct that a particular fact is, or should be, true about this new gadget we have defined. Thus, we go about trying to prove this fact. This process of attempting to prove that which we know is, or want to be, true helps us refine our definition. We can add to our definition whatever we need in order to prove our desired result. Then, we test our definition by going through this process again: that is, trying to prove a particular fact is true. Maybe we altered our definition too much, or maybe we need to add more to it. This process of testing out the definition continues until we are satisfied that the definition really does capture all the properties we want our new gadget to have.

Definitions in mathematics are extremely precise, and they need to be in order to prove results about them. It's not enough to just have an idea of what the concept is. For example, suppose I asked you to show that the sum of two odd numbers is even. You could test it out: $3 + 5 = 8$; $7+ 9 = 16$; etc. It seems true. But to really prove it, we need to show that it is true no matter which odd numbers we picked! Thus, we need to know the definitions of odd and even numbers.

Recall that **integers** are the positive and negative counting numbers and zero: …, -3, -2, 1, 0, 1, 2, 3,…

We say an integer x is **odd** if x can be written as $x = 2n +1$ for an integer n. We say that an integer y is **even** if y can be written as $y = 2m$ for an integer m. Now we can prove our result: Let a and b be odd numbers. Then $a = 2n +1$ and $b = 2m + 1$, for some integers n and m. Then we have:

$$a + b = 2n + 1 + 2m + 1 = 2n + 2m + 2 = 2(n + m + 1)$$

Since $n + m + 1$ is just a sum of integers, it is an integer itself, so $2(n + m + 1)$ is even by our definition.

Ken: Will you please describe a mathematical result that you proved?

Alissa: In addition to the research that I do related to what I did in my PhD dissertation, I'm also interested in the relationships between math and music and using mathematics to solve puzzles and games. So, recently, together with a colleague, we analyzed a puzzle called the *KO*

Labyrinth, which is a spherical maze that moves like a Rubik's Cube. Using many different mathematical fields including linear algebra, graph theory, and probability, we were able to prove numerous facts about the puzzle, including finding the shortest route through the maze.

Ken: Could you please describe in some detail one technique you have found to be helpful towards problem solving?

Alissa: I find that patience, tenacity, taking breaks, thinking about the problem from different viewpoints, and having lots of tools in my toolbox of skills are all helpful. When I got to college, I found that I couldn't solve every problem immediately, as I could for many of my high school homework problems, which was initially very frustrating! But then I realized I needed to cut myself some slack and have more reasonable expectations of myself. It's not reasonable to expect to be able to solve every problem immediately after reading it! So, I quickly learned that I needed to be more patient, which required that I start my homework early to give my brain time to figure out the solutions before the assignment was due. Also, I found that when I got stuck, the best thing I could do was go practice (the clarinet). Doing something completely different let my subconscious work on the problem and often when I got back to my dorm room, I had a new idea or new approach to the problem. Finally, I like looking at problems through different mathematical goggles: sometimes I wear my geometry/topology glasses and think about the problem geometrically and draw lots of pictures and other times I wear my algebra glasses and assign variables and scribble down equations. What works best for me is switching back and forth between these two vantage points because I often see something one way that I wouldn't have seen the other (e.g. a picture I've drawn might give me some intuition to a formula that I can use or prove).

Ken: What characteristics do you think you possess that enable you to be a successful mathematician?

Alissa: I love mathematics! I think that this is really the main reason one becomes a mathematician. Mathematics can be difficult and frustrating and one often spends more time being "stuck" than not. So, you have to really love it. Also, I have a strong work-ethic and focus, both of which help, I think.

Ken: Do you have any advice for how best to learn and understand mathematics?

Alissa: Patience, a willingness to ask questions, curiosity, and lots of practice! I often compare learning mathematics to learning a sport or instrument. The basketball player spends hours practicing free-throws and the musician spends equally as much time learning and perfecting their scales. A budding mathematician must do the same and really perfect their techniques by doing lots and lots of problems! Also, at the end of the day, we need time to sit alone and process the information for ourselves and make connections in our brains to other mathematics we already know and understand.

Ken: Last year, you became the Associate Director of Diversity and Education at MSRI. What are your goals in this position? What problems do you hope to address?

Alissa: I'm very passionate about supporting students, teachers, and faculty from under-represented groups with the goal of broadening participation in mathematics. Unfortunately, there are not many women or people of color who pursue higher degrees in mathematics. At the undergraduate level, about 50% of mathematics degrees are earned by women, but that drops down to about 20% when we talk about advanced degrees. The numbers are even more dire for men and women of color (African American, Latino/a, Native Americans). In my role at MSRI, I work together with the Director and Deputy Director on programs, activities, and initiatives to address these concerns and hopefully begin to make improvements.

Ken: You're affiliated with an educational program called Pathways. What is Pathways and how can students participate?

Alissa: Pathways is a mathematics outreach program for LA County schools. I am part of a team of mathematics faculty members who teach at LA County colleges and universities and who visit K-12 classrooms to share our love and passion for mathematics. We give presentations on various topics that do not appear in the usual school curriculum. Students can participate by asking their teacher to request that one of us visit their classroom![1]

Ken: When you're not doing math, what do you like to do?

Alissa: I'm also a musician, so I play in a local wind ensemble weekly. I have played the clarinet since the 5th grade. I also enjoy being outdoors, so I love running, biking, swimming, and anything else my friends suggest. I like baking, spending time with my family and friends, reading, and making my own greeting cards.

Ken: Just out of curiosity, how did you get a Bacon number of 2?

Alissa: Well, I'm not sure how legitimate it is! It's more of a joke. When I was in high school, my marching band was in the movie "Higher Learning." So, I was technically in a movie, even if I didn't have an individual role.[2]

Definitions in mathematics are extremely precise, and they need to be in order to prove results about them.

Ken: I'll have to check out that movie! Do you have any parting advice for our readers?

Alissa: Don't be shy about asking lots of questions! Mathematicians love talking about what they do and explaining their work to others. Get involved in mathematics outside of your school: Form a math club, join a math circle, attend a Sonya Kovalevsky Day, Expanding your Horizons Conference, or Julia Robinson Mathematics Festival. Participate in the AWM (Association for Women in Mathematics) essay contest[3]. Attend a summer program/camp in mathematics[4].

Identify role models or mentors for yourself. Take advantage of the mentor program with Girls' Angle and/or the AWM Mentor Network[5].

Ken: Thank you for this interview!

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[1] If you are interested in having Pathways visit your class or math club, or if you have any questions about Pathways, please send an email to pathways@math.hmc.edu.

[2] If you were in a movie with the actor Kevin Bacon, your Bacon number is 1. If you weren't in a movie with Kevin Bacon, but were in one with someone with a Bacon number of 1, your Bacon number is 2, etc. The actress Kari Wuhrer, who was also in *Higher Learning*, appeared in *The Air I Breathe*, with Kevin Bacon.

[3] sites.google.com/site/awmmath/programs/essay-contest

[4] www.ams.org/programs/students/high-school/emp-mathcamps

[5] sites.google.com/site/awmmath/programs/mentor-network